Algebra

Elementary algebra is an area of mathematics where numbers and quantities called variables are represented by letters and symbols in expressions and equations.

An equation is a mathematical statement containing an equals sign. One number is represented by an unknown variable. To solve an equation, the value of this variable must be found.

Exponent

Coefficients **Variable** **Constants**

$$5x^2 + 2x - 8 = 16$$

Terms **Operators**

Expression
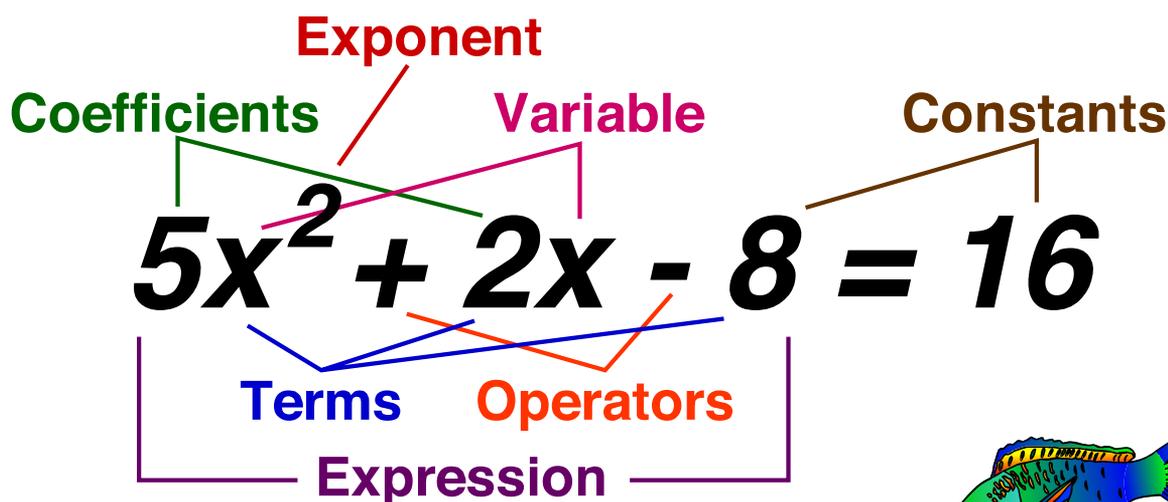
Parts of an equation

Variable • varying quantity represented by a letter or symbol, e.g. x

Constant • a fixed quantity that does not vary, e.g. a number

Coefficient • a number which multiplies a variable, e.g. $5x$

Exponent • shows the number of times a variable or number is multiplied by itself, e.g. $y^4 = y \times y \times y \times y$

Operator • a symbol indicating what operation must be done, e.g. $+ - x ÷$

Term • one part of an expression which may be a number, a variable or a product of both, e.g. $5x^2$ $4xy$ 12

Expression • one or a group of terms. May include variables, constants, operators and grouping symbols e.g. $3(x + y) - 8 + y^2$
An algebraic expression must contain at least one variable.

Variables

A variable is a letter or symbol representing a varying quantity, for example, *n* in *10 + n*.

A variable is the opposite to a constant - a variable can change (vary) but a constant remains the same.

A variable may be represented by any letter of the alphabet.

Variables in expressions

EXAMPLES: To evaluate (work out) each expression if *z = 10*.

$6 + z$
$= 6 + 10$
$= 16$

$20 - z$
$= 20 - 10$
$= 10$

$2z$
$= 2 \times 10$
$= 20$

z^2
$= 10 \times 10$
$= 100$

Variables in equations

EXAMPLE: A variable may be given a number e.g. *n = 3* so we can work out another variable e.g. *t*

$t = 10 + n$
$t = 10 + 3$
$t = 13$

$t = 10 - n$
$t = 10 - 3$
$t = 7$

$t = 10n^2$
$t = 10 \times 9$
$t = 90$

Terms

A term is one part of an expression which may be a number, a variable or a product of both.

An expression is one or a group of terms and may include constants, operators and grouping symbols.

Expression	Terms
$2a + b^2$	$2a$ and b^2
$4x^3 + 3xz - 5$	$4x^3$, $3xz$ and 5
$9x^3 + 5x^3 + x + 16$	$9x^3$, $5x^3$, x and 16

Some types of terms

▶ **variable term**
  • a term that contains a variable.

▶ **constant term**
  • a term that has a fixed value and does not contain a variable.

▶ **like terms**
  • terms that are exactly alike or the same except for their numerical coefficients, e.g.

  $2y^2$, $10y^2$, $-2y^2$ ... all terms have y^2 with a coefficient.

  $2y$, $10y$, y, $-y$ ... all terms have a single variable, y.

  20, -5, 6, 0.6 ... all terms are constants.

▶ **unlike terms**
  • terms that are not like terms.
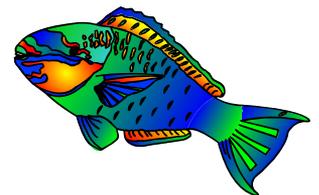
Expressions

An expression is one or a group of terms and may include variables, constants, operators and grouping symbols
e.g. 3(x + y) - 8 + 2y

Some parts of an expression.

Grouping symbols   **Constant**

Variables

$$3(x + y) - 8 + 2y$$

Terms   **Operators**

Expression
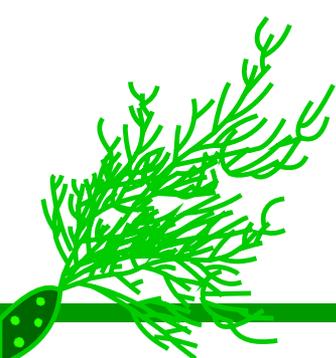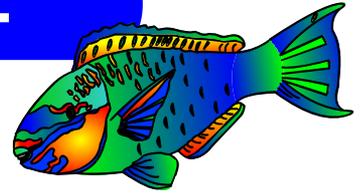
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= ≠   **expressions DO NOT contain equality or inequality signs**   **> <**

An equation uses an equals sign between two expressions.
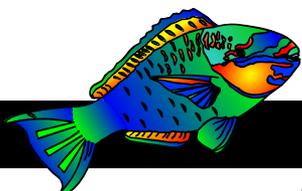
Equations

An equation is a mathematical statement containing an equals sign. One number is represented by an unknown variable. To solve an equation, the value of this variable must be found.

Parts of an equation

Exponent

Coefficients   **Variable**   **Constants**

$$5x^2 + 2x - 8 = 16$$

Terms   **Operators**

Expression
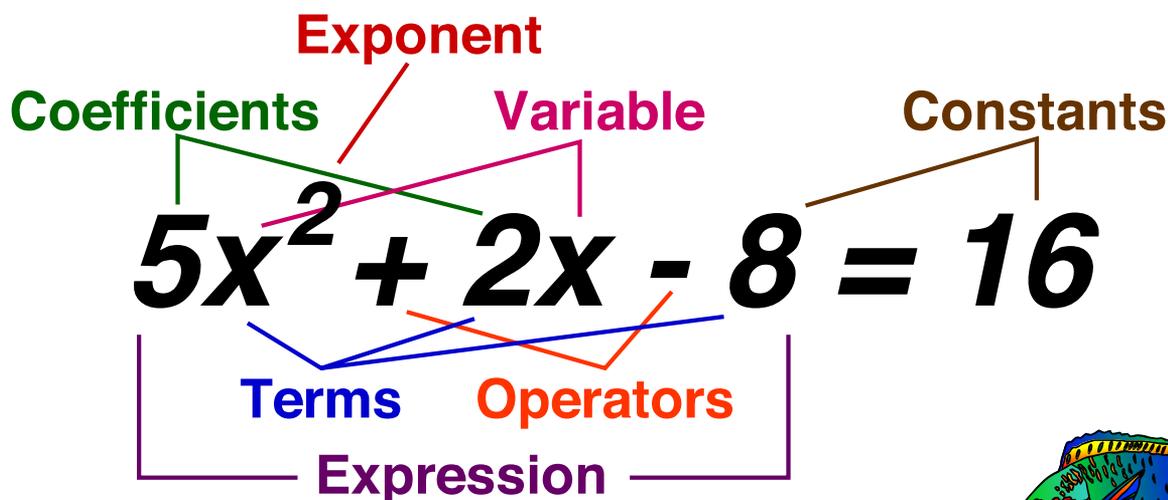
Solving equations - the basic rule.

Whatever operation is done to one side of an equation, the same operation must be done to the other side.

One step equation, e.g.
- divide both sides by 4

$4x = 12$
$x = 3$

Two step equation, e.g.
- add 10 to both sides
- divide both sides by 6

$6y - 10 = 20$
$6y = 30$
$y = 5$

Equations with brackets, e.g.
- remove the grouping symbols
- subtract 6 from both sides
- divide both sides by 3

$3(x + 2) = 24$
$3x + 6 = 24$
$3x = 18$
$x = 6$

Exponents, indices, powers, orders

Exponents, indices, powers and orders all mean the same thing.

Exponent or index or power or order

An exponent (index, power or order) is a small number placed to the upper-right of a number which shows how many copies of the base number are multiplied together.

exponent, index, power or order

$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$

base   expanded   value

Examples

Exponent, Index or Power Base	Expanded	Value	Read as
3^2	3 x 3	9	three **squared** OR three **to the power of** two
5^3	5 x 5 x 5	125	five **cubed** OR five **to the power of** three
10^4	10 x 10 x 10 x 10	10 000	ten **to the power of** four
4^5	4 x 4 x 4 x 4 x 4	1024	four **to the power of** five

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1000	100	10	1	0.1	0.01	0.001
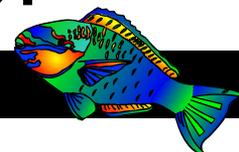
Order of operations

In mathematics, the order of operations is the order in which operations should be done.

If you do the operations in the wrong order – you will get the wrong answer!!

Often acronyms such as PEMDAS, BIDMAS or BODMAS are used to help remember the sequence.

PEMDAS

2

√4

parentheses

exponents, roots

multiplication

division

addition
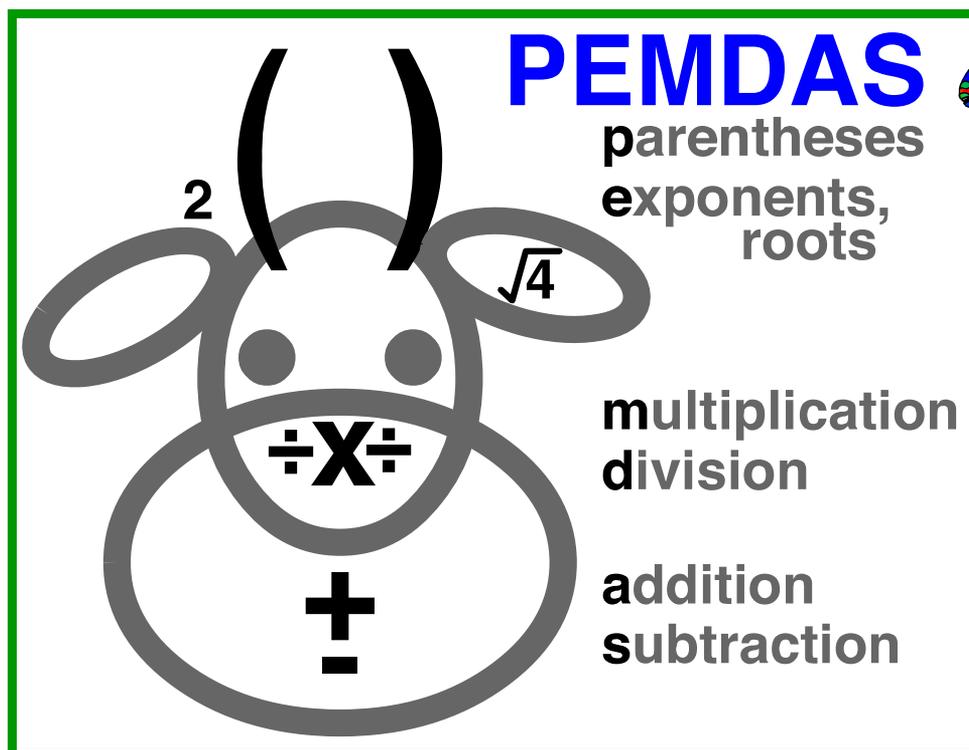
subtraction

PEMDAS

1. **P** arentheses ( ) or {} or [ ] , brackets
2. **E** xponents (indices, orders, powers), roots
3. **M** ultiplication (times)
   D ivision (divided by)

   Multiplication and division have equal precedence.
4. **A** ddition (plus)
   S ubtraction (minus)

   Addition and subtraction have equal precedence.

RULE: Always work left to right.

Operations Properties

Addition

Associative property

$(a + b) + c = a + (b + c)$ $(4 + 2) + 1 = 4 + (2 + 1)$

Commutative property

$a + b = b + a$ $4 + 2 = 2 + 4 = 6$

Additive identity property of 0

$a + 0 = 0 + a = a$ $4 + 0 = 0 + 4 = 4$

Multiplication

Associative property

$(a \times b) \times c = a \times (b \times c)$ $(4 \times 2) \times 1 = 4 \times (2 \times 1)$

Commutative property

$a \times b = b \times a$ $4 \times 2 = 2 \times 4 = 8$

Multiplicative identity property of 1

$a \times 1 = 1 \times a = a$ $4 \times 1 = 1 \times 4 = 4$

Zero product property

$a \times b = 0$ either $a = 0$, $b = 0$ or both a and $b = 0$

Distributive property of multiplication over addition

$a \times (b + c) = a \times b + a \times c$ $4 \times (2 + 1) = 4 \times 2 + 4 \times 1$

Inverses

Additive inverses

$a + (-a) = (-a) + a = 0$ $4 + (-4) = (-4) + 4 = 0$

Multiplicative inverses

$a \times 1/a = 1/a \times a = 1$ $4 \times 1/4 = 1/4 \times 4 = 1$

if $a \neq 0$